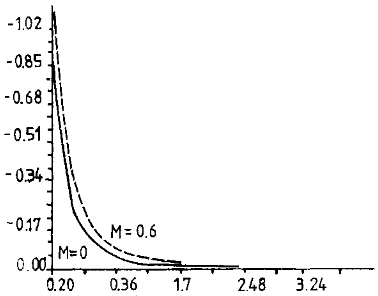

Fig. 3 The N_1 for a symmetrical Joukowski airfoil.

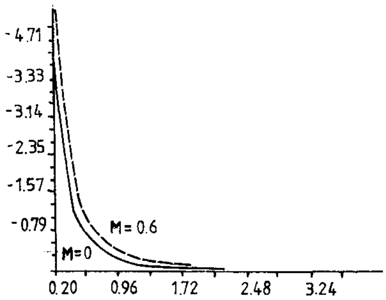

Fig. 4 The N_2 for a symmetrical Joukowski airfoil.

we obtain

$$\bar{H}_i = \frac{2\beta D}{m+1}\sum_{\gamma=1}^{m}\frac{1-\zeta_\gamma^2}{(x_i-\zeta_\gamma)^2+4\beta^2D^2}$$

$$-\frac{4\beta D}{2m+1}\sum_{\alpha=1}\frac{(1-\zeta_\gamma)\zeta_\gamma}{(x_i-\zeta_\alpha)^2+4\beta^2D^2} \tag{18}$$

In this case, the dependence of N_1 and N_2 on D is given in Figs. 3 and 4. The lift is negative, i.e., the resulting force is toward the ground.

The present theory, linear with respect to the thickness parameter ϵ, is not valid in the close neighborhood of the ground where D becomes a small parameter too. In this region, a new theory is necessary.

Acknowledgment

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References

[1]Panchenkoff, A. N., *Hydrodynamics of the Wing Under Water*, Naukova Dumka, Kiev, 1965 (in Russian).

[2]Panchenkoff, A. N., *Theory of the Potential of Accelerations*, Nauka, Novosibirsk, 1975 (in Russian).

[3]Plotkin, A., and Kennel, C., "Thickness-Induced Lift on a Thin Airfoil in Ground Effect," *AIAA Journal*, Vol. 19, No. 11, 1981, pp. 1484-1486.

[4]Plotkin, A., and Dodbele, S., "Slender Wing in Ground Effect," *AIAA Journal*, Vol. 26, No. 4, 1988, pp. 493-494.

[5]Iuhimenko, A. I., "Influence of Airfoil Shape on the Aerodynamical Characteristics of the Wing near the the Ground, in High Speed Hydrodynamics," Vol. 2, Naukova Dumka, Kiev, 1966 (in Russian).

[6]Dragos, L., "Method of Fundamental Solutions in Plane Steady Linear Aerodynamics," *Acta Mechanica*, Vol. 47, No. 3, 1983, p. 277.

[7]Dragos, L., *Method of Fundamental Solutions*. A Novel Theory of Lifting Surface in a Subsonic Flow," *Arch. Mech.*, Vol. 35, 1983, p. 575.

[8]Mouskhelishvili, W. I., "Integral Singular Equations," Gostehizdat, Moscow, 1962 (in Russian).

[9]Stark, V. J. E., "A Generalized Quadrature Formula for Cauchy Integrals," *AIAA Journal*, Vol. 9, No. 9, 1971, pp. 1854-1855.

Optimum Hypersonic Airfoil with Power Law Shock Waves

B. A. Wagner*
Rensselaer Polytechnic Institute,
Troy, New York 12180

Introduction

In the present Note, the flowfield over a class of two-dimensional lifting surfaces is examined from the viewpoint of inviscid, hypersonic small-disturbance theory (HSDT). It is well known that a flowfield in which the shock shape $S(x)$ is similar to the body shape $F(x)$ is only possible for $F(x) = x^k$ and the freestream Mach number $M_\infty = \infty$. This self-similar flow has been studied for several decades since it represents one of the few existing exact solutions of the equations of HSDT. Detailed discussions are found, for example, in papers by Cole,[1] Mirels,[2] Chernyi,[3] and Gersten and Nicolai,[4] but they are limited to convex body shapes, that is, $k \leq 1$. The only study of concave body shapes was attempted by Sullivan[5] where only special cases were considered. The method used here shows that similarity also exists for concave shapes, and a complete solution of the flowfield for any $k > 2/3$ is given. The effect of varying k on $C_L^{3/2}/C_D$ is then determined, and an optimum shape is found. Furthermore, a wider class of lifting surfaces is constructed using the streamlines of the basic flowfield and analysed with respect to the effect on $C_L^{3/2}/C_D$.

We neglect viscous effects and assume boundary layers to be thin and attached to the surface. The surfaces are considered to correspond to the lower compression surface of a two-dimensional wing. Since the pressure difference across the shock induced by this surface is of higher order than that of the shock induced by the upper expansion surface, we neglect the contribution of the upper surface to the lift or drag.

Similarity Solution

This section is a formulation of our problem in the framework of hypersonic small-disturbance theory. If we substitute the scaled variables $y = \bar{y}/\delta$ and $x = \bar{x}$, with δ = thickness ratio, together with the asymptotic representations for velocity, pressure, and density into the equations of motion and neglect $O(\delta^2)$ terms, we obtain a reduced problem with the longitudinal momentum equation uncoupled from the rest of the problem. This longitudinal momentum equation can later be determined using the Bernoulli equation.

For a slender airfoil we write for the body surface $\bar{y} = \delta F(x)$ with associated shock shape $\bar{y} = \delta S(x)$. See Fig. 1.

Next, we change the (x,y) coordinate system to the (x,ψ) coordinate system, where ψ is the stream function. See Fig. 2. We further change from (x,ψ) to (x,ξ) coordinates, where ξ is the shock location $x = \xi$, i.e., the x location where an incoming streamline crosses the shock. See Figs. 2 and 3. Note that ψ and ξ are related by $\theta(\xi)\partial/\partial\psi = \partial/\partial\xi$, where $\theta(\xi) = \mathrm{d}S(\xi)/\mathrm{d}\xi$ can be determined by a separation of variables argument together with the continuity and momentum equation as $\theta(\xi) = ka\xi^{k-1}$. Also note that now the density can be written in terms of the pressure using the entropy equation and the shock conditions. Finally, we obtain for our basic problem

Continuity:

$$\frac{c^2\theta(\xi)^{\frac{2}{\gamma}+1}}{p^{*\frac{1}{\gamma}}}\frac{\partial p^*}{\partial x} + \frac{\partial v^*}{\partial \xi} = 0 \tag{1}$$

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*Graduate Student, Applied Mathematics.

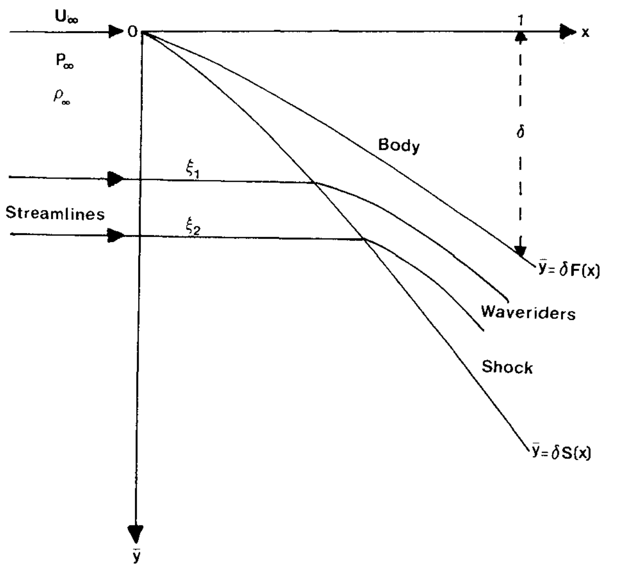

Fig. 1 Domain of boundary value problem (BVP) in dimensional coordinates.

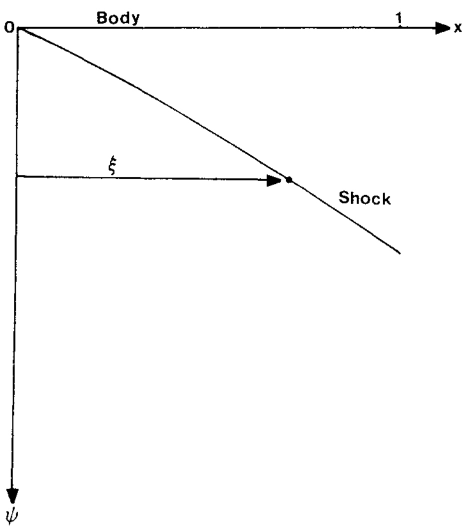

Fig. 2 Domain of boundary value problem (BVP) in y,x coordinates.

Momentum:

$$\frac{\partial p^*}{\partial \xi} + \theta(\xi)\frac{\partial v^*}{\partial x} = 0 \tag{2}$$

Shock conditions:

$$p^*(\xi,\xi) = p_S^* = \theta^2(\xi) \tag{3}$$

$$v^*(\xi,\xi) = v_S^* = \theta(\xi) \tag{4}$$

Boundary condition:

$$v^*(x,0) = \frac{\gamma+1}{2}\,\frac{\mathrm{d}F(x)}{\mathrm{d}x} \tag{5}$$

where $c^2 = (\gamma-1)/2\gamma$, $v = [2/(\gamma+1)]v^*$, $p = [2/(\gamma+1)]p^*$.

For the similarity solution we have

$$p^*(x,\xi) = k^2a^2x^{2k-2}\eta^{2k-\frac{2}{\gamma+1}}R(\eta) \tag{6}$$

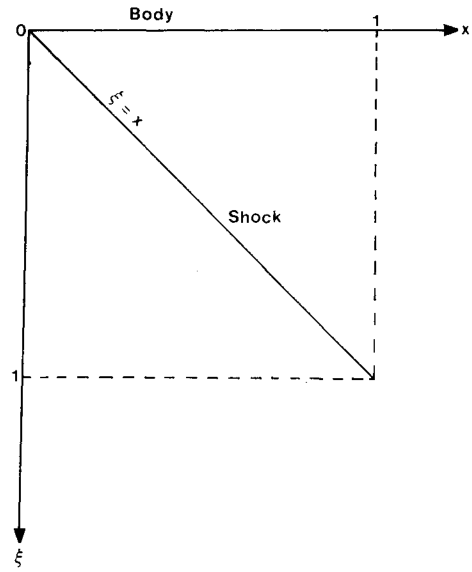

Fig. 3 Domain of boundary value problem (BVP) in ξ,x coordinates.

$$v^*(x,\xi) = kax^{k-1}\eta^{k-\frac{2}{\gamma+1}}U(\eta) \tag{7}$$

where $\eta = \xi/x$ is the similarity variable. Therefore we obtain, together with the shock conditions, Eqs. (3) and (4), $R(1)=1$ and $U(1)=1$. The boundary condition, Eq. (5), can be used to determine the constant a. If we substitute Eqs. (6) and (7) into Eqs. (1) and (2), we obtain

Continuity:

$$-\frac{2\gamma}{\gamma+1}\,\frac{c^2}{R^{\frac{1}{\gamma}}} - \frac{c^2}{R^{\frac{1}{\gamma}+1}}\eta\frac{\mathrm{d}R}{\mathrm{d}\eta} + \left(k-\frac{2}{\gamma+1}\right)U + \eta\frac{\mathrm{d}U}{\mathrm{d}\eta} = 0 \tag{8}$$

Momentum:

$$\left(2k-\frac{2}{\gamma+1}\right)R + \eta\frac{\mathrm{d}R}{\mathrm{d}\eta} - \frac{\gamma-1}{\gamma+1}U + \eta\frac{\mathrm{d}U}{\mathrm{d}\eta} = 0 \tag{9}$$

The initial value problem we obtained can be solved numerically using a Runge-Kutta method, where we are interested in the cases where $k>1$. Note that the special case of the Newtonian limit $\gamma=1$ can be solved completely analytically.

Evaluation of $C_L^{3/2}/C_D$

At first we will study the case where $\xi=0$, which is the case of the original power law shape. Observe that as $\eta\to 0$ we find that

$$R(\eta) = c_1\eta^{\frac{2}{\gamma+1}-2k} \tag{10a}$$

$$U(\eta) = c_0\eta^{\frac{2}{\gamma+1}-k} \tag{10b}$$

The coefficients c_0 and c_1 are determined using Eqs. (8) and (9). From the definition of the lift and drag coefficients and Eqs. (6) and (7), we obtain the following formula:

$$\frac{C_L^{3/2}}{C_D} = \sqrt{\gamma+1}\;\frac{3k-2}{(2k-1)^{3/2}}\sqrt{\frac{c_1}{c_0}} \tag{11}$$

We find that a maximum value of $C_L^{3/2}/C_D = 1.569$ is attained at $k=1.13$ for $\gamma = 7/5$. This result agrees with a result by Cole and Aroesty[6] who suggested that body shapes that are slightly more concave than a flat plate have superior performance.

Next, we wish to investigate the behavior of $C_L^{3/2}/C_D$ for $\xi \neq 0$. The underlying idea for constructing a wider class of lifting surfaces is to use the streamlines of our basic flowfield as the elements of the surface. Then the lifting surface is formed by those streamlines that penetrate the basic shock surface through the points on the leading-edge curve. See Fig. 1. Let us now define the lift and the drag coefficients as functions of ξ:

$$C_{L(\xi)} = -\frac{4}{\gamma+1} k^2 a^2 \delta^2 \xi^{2k-1} \int_1^{\frac{\xi}{\xi+1}} \eta^{-\frac{2}{\gamma+1}} R(\eta)\, d\eta \tag{12}$$

The integral in the last equation can be found by using the momentum equation. Hence,

$$C_{L(\xi)} = \frac{4}{\gamma+1} k^2 a^2 \delta^2 \frac{1}{2k-1} (\xi+1)^{2k-1} \left(\frac{\xi}{\xi+1}\right)^{2k-\frac{2}{\gamma+1}} \times \left[R\left(\frac{\xi}{\xi+1}\right) - U\left(\frac{\xi}{\xi+1}\right)\right] \tag{13}$$

Similarily, we have for the drag coefficient

$$C_{D(\xi)} = -\frac{8}{(\gamma+1)^2} k^3 a^3 \delta^3 \xi^{3k-2} \int_1^{\frac{\xi}{\xi+1}} \eta^{\frac{\gamma-3}{\gamma+1}} R(\eta) U(\eta)\, d\eta \tag{14}$$

Using momentum and continuity, we can integrate and obtain

$$C_{D(\xi)} = -\frac{8}{(\gamma+1)^2} k^3 a^3 \delta^3 \frac{1}{3k-2} (\xi+1)^{3k-2} \left(\frac{\xi}{\xi+1}\right)^{3k-\frac{4}{\gamma+1}} \times \left[U\left(\frac{\xi}{\xi+1}\right) R\left(\frac{\xi}{\xi+1}\right) - \frac{U\left(\frac{\xi}{\xi+1}\right)^2}{2} - \frac{1}{2} R\left(\frac{\xi}{\xi+1}\right)^{\frac{\gamma-1}{\gamma}} \right] \tag{15}$$

Finally, we obtain for the formula for a general two-dimensional waverider

$$\frac{C_{L(\xi)}^{3/2}}{C_{D(\xi)}} = \sqrt{\gamma+1}\, \frac{3k-2}{(2k-1)^{3/2}} \sqrt{\xi+1} \times \frac{\left\{ \eta^{2k-\frac{2}{\gamma+1}} [R(\eta) - U(\eta)] \right\}^{3/2}}{\left\{ \eta^{3k-\frac{4}{\gamma+1}} \left[U(\eta) R(\eta) - \frac{U(\eta)^2}{2} - \frac{1}{2} R(\eta)^{\frac{\gamma-1}{\gamma}} \right] \right\}} \tag{16}$$

at $\eta = \xi/\xi+1$. An examination of the behavior of $C_{L(\xi)}^{3/2}/C_{D(\xi)}$ shows an increase of the maxima as ξ increases whereas the k where the maxima are attained also increase. The highest maximum value of $C_{L(\xi)}^{3/2}/C_{D(\xi)}$ is the limiting case $\xi \to \infty$ and $k_{max} \to \infty$ that corresponds to the body shape supporting exponential shock shapes. The limiting value is $C_{L(\xi)}^{3/2}/C_{D(\xi)} = 1.5795$. This special case was worked out earlier by Cole and Areosty.[7]

Concluding Remarks

Details of above investigations can be found in Wagner.[8] The analysis is part of a study of optimum lifting surfaces using HSDT and will be used, in a subsequent paper, to design three-dimensional waveriders supported by two-dimensional flowfields. This represents a generalization of the idea by Nonweiler[9] to design three-dimensional inverted-V wings supported by the two-dimensional flowfield generated by a flat plate.

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References

[1]Cole, J. D., "Newtonian Flow Theory for Slender Bodies," *Journal of the Aeronautical Sciences,* Vol. 24, No. 6, 1957, pp. 448–455.

[2]Mirels, H., "Hypersonic Flow Over Slender Bodies Associated with Power-Law Shocks," *Advances in Applied Mechanics,* Vol. 7, 1962.

[3]Chernyi, G. G., *Introduction to Hypersonic Flow,* Academic, New York, 1961.

[4]Gersten, K., and Nicolai, D., *Die Hyperschallströmung um schlanke Körper mit Konturen der Form $\bar{R} = \bar{x}^n$,* Deutsche Luft- and Raumfahrt Forschungsbericht 64–19, July, 1964.

[5]Sullivan, P. A., "Inviscid Hypersonic Flow on Cusped Concave Surfaces," *Journal of Fluid Mechanics,* Vol. 24, No. 1, 1966.

[6]Cole, J. D., and Aroesty, J., "Optimum Hypersonic Lifting Surfaces Closer to Flat Plates," *AIAA Journal,* Vol. 3, Aug. 1965, pp. 1520–1522.

[7]Cole, J. D., and Aroesty, J., "Hypersonic Similarity Solutions for Airfoils Supporting Exponential Shock Waves," *AIAA Journal,* Vol. 8, No. 2, 1970, pp. 308–315.

[8]Wagner, B. A., "Hypersonic Similarity Solutions for Airfoils Supporting Power Law Shock Waves," Rensselaer Polytechnic Institute, Troy, NY, RPI Math. Rept. 168, July 1988.

[9]Nonweiler, T. R. F., "Delta Wings of Shapes Amenable to Exact Shock Wave Theory," *Journal of the Royal Aeronautical Society,* Vol. 67, No. 625, Jan. 1963.

Effect of Slotting on the Noise of an Axisymmetric Supersonic Jet

Anjaneyulu Krothapalli*
Florida A&M University/Florida State University, Tallahassee, Florida 32316

James McDaniel†
University of Virginia, Charlottesville, Virginia 22903
and
Donald Baganoff‡
Stanford University, Stanford, California 94305

Introduction

THE purpose of this Note is to report the results of an experiment that demonstrates a simple concept for noise reduction in a supersonic jet exiting from a converging axisymmetric nozzle. The basic axisymmetric nozzle exit geometry is modified by the addition of fingers, as shown in Fig. 1.

It is well-known that the structure of a choked underexpanded jet has features different from those of subsonic and ideally expanded supersonic jets. These features include dis-

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*Professor and Chairman, Department of Mechanical Engineering, College of Engineering. Member AIAA.

†Associate Professor, Department of Aerospace and Mechanical Engineering. Member AIAA.

‡Professor, Department of Aeronautics and Astronautics. Member AIAA.